

- I. *An Account of Observations made on Board the Chatham-Yacht, August 30th and 31st, and September 1st, 1732, in pursuance of an Order made by the Right Honourable the Lords Commissioners of the Admiralty, for the Trial of an Instrument for taking Angles, described in Philosophical Transactions, Numb, 420. By John Hadley, Esq; Vice-President of the Royal Society.*

IN *May, 1731*, I communicated to the *Society* the Description of a new Instrument for taking Angles, and produced a Specimen of an Instrument made accordingly. Several of the Gentlemen to whom it was shewn, as well then as at other times, entertained a favourable Opinion of the Probability of its Usefulness, particularly our worthy Vice-President *Dr. Edmund Halley*, *Astr. Reg.* and the Reverend *Mr. James Bradley*, *Astr. Pr. S.* not only expressed their Desire that Trial should be made of it at Sea, but promised the Favour of their Company and Assistance on that Occasion.

The Instrument produced at the *Society* was made of Wood, according to *Fig. IV. Transact. N^o 420.* of the forementioned Description, and was intended chiefly for taking Altitudes of the Sun, Moon and Stars, from the visible Horizon, either forwards or backwards; I therefore procured another to be made of Brass by *Mr. F.*

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Sisson,

Siffon, for taking the Distance of any kind of Objects. It is supported by a single Stem skrewed on to it on the under Side, the lower End of which may rest on the Ground, to ease the Observer of the Weight of the Instrument. This Stem is also made to lengthen or shorten, by which Means the Instrument is brought to the proper Height for any Observer's Eye, either standing or sitting. Instead of a Ball and Socket, it has two circular Arches fixed on its Back, by which it is readily set to any Position which the Situation of the Objects may require.

The Right Honourable the Lords Commissioners of the Admiralty having been pleased to order the *Chatbam-Yacht* for the Trial of the said Instrument, and to give Directions to Mr. *James Young*, Master Attendant at *Chatbam*, a Gentleman well skill'd in Navigation, to be present at the Trial, my two Brothers and Self went on Board accordingly *Wednesday, August* the 30th, being favoured with the Company (besides the two forementioned Gentlemen) of the Reverend Sir *Robert Pye*, Bart. and *Robert Ord*, Esq; Members of this *Society*. We met Mr. *Young* at *Sheernefs* the next Day, who accompanied us down about three Leagues below the *Nore*, near the *Spile-Sand*, and was on Board on *Friday, September* the 1st, when we lay by there, and the several Altitudes of the Sun were taken as it approached the Meridian from about Ten of the Clock 'till Noon.

The Observations were as follow.

August the 30th, near Midnight, Mr. *Bradley* observed the Distance of *Lucida Iyræ* from *Cor Aquilæ* by the Brass Instrument off *Gravefend* in still Water,

34° 13' 50'

The same repeated was

34 13 15

The Error of the Instrument in that Place is 23" to be subtracted.

The Distance of those Stars, according to Mr. *Flamsteed*, is

34° 11' 50"

Which by the Refraction is reduced to

34 11 10

August the 31st, about 10^h 30', Mr. *Bradley* observed the Distance of *Capella* from the *North Pointer* in the *Great Bear's* Back, by the same Instrument, while we lay at Anchor in the Mouth of the *Medway* near *Sheerness*, the Wind blowing hard at North East,

49° 14' 00" +

Or 49 15 00

Mr. *Bradley* and my self making a small Difference in numbring the Angle mark'd by the Index.

The Error of the Division of the Instrument there is 30" to be added.

The Distance of those Stars, according to Mr. *Flamsteed*, is

49° 16' 00"

By the Refraction reduced to

49 14 20

Clouds coming up prevented the repeating this Observation, nor had we any Opportunity of making any others of this kind.

Altitudes of the Sun observed by Mr. *Bradley*, lying at Anchor in the Mouth of the *Medway*, *August* the 31st, Afternoon, the Wind at North-East, a fresh Gale, by the Wooden Instrument forwards. The Watch by the Mean of the Observations appeared to be about 8' 45" too slow; the visible Horizon being supposed 3' 30" depressed below the true by the Height of the Observer's Eye above the Surface of the Water, amounting to about 8 or 9 Feet.

Time by Watch.	True Time.	True Alt. of Sun's upper Limb from the visible Horizon.	Refract. add.	Appr. Alt. of Sun's upper Limb from the visible Horizon.	Alt. of the Sun's upper Limb observed.	Errors of vision of the Sun's upper Limb, corrected.	Observed Alt. of the Sun's upper Limb corrected.	Error of Observation.
5 11 50	5 20 35	50 31	4	55 25	57 00	2 15	54 45	40
16 30	25 15	07 00	5	12 17	13 30	2 15	11 15	02
18 20	27 05	49 52	5	55 19	57 30	3 00	54 30	49
21 20	30 05	21 51	5	27 35	30 00	3 00	27 00	35
28 5	36 50	18 44	6	25 12	27 30	2 00	25 30	18
30 55	39 20	55 22	6	02 08	05 00	2 15	02 45	37
32 25	41 10	38 13	7	45 16	48 00	2 15	45 45	29
36 30	45 15	00 00	7	07 40	10 00	3 15	07 45	05
38 37	47 22	40 11	8	48 14	51 00	2 15	48 45	31
40 35	49 20	21 50	8	30 14	34 00	2 15	31 45	31
42 34	51 19	03 14	8	12 04	15 00	3 00	12 00	04
43 50	52 35	51 24	9	00 51	03 30	3 00	00 30	1

Altitudes

Altitudes of the Sun, observed *September* the 1st, before Noon, under Sail from *Sheerneys* towards the *Spile-Sand*, with the Tide of Ebb, the Wind blowing hard at North-East, by the Wooden Instrument forward. The second Speculum being removed by some Accident from its due Position, so as to increase the Angles observ'd about one Degree three Minutes and a half, as appeared by the first Observations of the Afternoon of the same Day, made with the same Instrument, in the same manner, while we continued lying-by near the *Spile*; and that Degree and three Minutes and a half are added to the Errors of the Divisions of the Instrument in the seventh Column. While these Observations were making, the Yacht steered at first chiefly East, sometimes South-East, afterwards stood to the North-East, towards the *Swin*. The Time of the Watch was regulated by some of the later Observations made when we were most Eastward, and this was probably the Cause why the first Altitudes, which were taken while we were more Westerly, fall so much short of the Computations, the Difference decreasing gradually as we advanced towards the East.

Altitudes observed by Mr. Bradley.

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Time by Watch.	True Time.	True Alt. of the Sun's lower Limb from the visible Horizon.	Refract- ion, add.	App ^d Alt. of the Sun's lower Limb from the visible Ho- rizon.	Altitude of the Sun's lower Limb observed.	Errors of the In- strument	Observed Altitude of the Sun's lower Limb corrected.	Errors of Observa- tion.
7 09 15	18 15 15	05 39 3	15 15	08 54 16	09 00	1 5 45	03 15	5 39
11 44	20 44	28 13 3	11 11	31 24	33 00	1 5 45	27 15	4 09
13 58	22 58	45 23 5	08 08	48 31	49 00	1 5 45	43 15	5 16
14 55	23 55	56 43 3	05 05	59 48 17	02 00	1 5 45	56 15	3 33
16 33	25 33 16	11 47 3	02 16	14 49	18 00	1 5 45	12 15	2 34
18 04	27 04	25 31 2	59 59	28 30	32 00	1 6 30	26 15	2 15
23 54	52 54 17	18 00 2	50 17	50 18	25 00	1 6 30	19 30	1 20
25 38	34 38	33 31 2	47 47	56 18	43 00	1 6 00	37 00	10 42
28 25	37 25	58 23 2	43 18	01 06 19	07 00	1 6 00	01 00	0 06
30 44	39 44 18	19 04 2	40 40	21 44	28 00	1 5 00	23 00	1 16
34 21	45 21	51 10 2	35 35	53 45 20	00 00	1 4 30	55 30	1 45
36 24	45 24 19	09 18 2	33 19	11 51	16 00	1 4 30	11 30	0 21
38 44	47 44	29 54 2	30 30	32 24	38 00	1 4 30	33 30	1 06
40 50	49 50	45 27 2	28 28	47 55	52 00	1 4 30	47 30	0 25
42 00	51 00	58 41 2	26 20	01 07 21	04 00	1 4 30	59 30	1 37
45 34	54 34 20	29 51 2	22 22	32 13	35 00	1 4 30	30 30	1 43

uncertain

The same continued by Mr. John Hadley.

Time by Watch.	True Time.	True Alt. of the Sun's lower Limb from the visible Horizon.	Refract. on the visible Horizon.	App ^t Alt. of the Sun's lower Limb from the visible Horizon.	Altitude of the Sun's lower Limb observed.	Errors of the Instrument (subtract).	Observed Altitude of the Sun's lower Limb corrected.	Errors of Observation.					
7 52	18 01	18 21	28	21 2	15 21	30	36 22	36 00	1 4	30 21	31 30	1 0	54
54	18 03	18 45	43	2 13	47 56		52 00	52 00	1 4	30 30	47 30	0 0	26
55	40 04	57 25	2 12	59 37	23 04	00	30 00	30 00	1 4	30 30	59 30	0 0	07
58	22 07	22 22	20	34 2	09 22	22	43 30	43 00	1 5	15 22	24 45	1 2	02
8 02	51 11	51 51	58	46 2	06 23	00	52 24	52 00	1 5	15 15	58 45	2 0	07
09	19 18	19 23	53	26 2	00 55	26 25	00 00	00 00	1 5	30 23	54 30	0 0	56
13	10 22	10 24	25	35 1	57 24	27 32	32 00	32 00	1 5	30 24	26 30	1 0	02
14	45 23	45 55	38	43 1	56 40	59 45	00 00	00 00	1 5	30 30	39 30	1 0	09
16	55 25	55 55	56	24 1	55 58	19 26	03 00	03 00	1 5	30 30	57 30	0 0	49
19	05 28	05 25	14	34 1	53 25	16 27	22 00	22 00	1 5	30 25	16 30	1 0	03
22	57 51	57 46	14 1	51 48	05 48	05	52 00	52 00	1 5	30 30	46 30	1 0	35
25	05 34	05 26	03	35 1	49 26	05 24 27	10 00	10 00	1 5	30 26	04 30	0 0	54
26	43 35	43 16	50 1	48 18	08 18	08	22 00	22 00	1 5	30 30	16 30	1 0	38
28	20 37	20 29	54 1	47 51	41 51	41	35 00	35 00	1 5	30 30	29 30	2 0	11

Altitudes of the Sun, observed lying by near the *Spile*, *September* the 1st, before Noon, with the Wooden Instrument backward, the Wind continuing to blow hard, as before, at North-East. The Instrument when used for the back Observation was so adjusted, as to allow for a Dip of the visible Horizon of two Minutes and a half; consequently that Dip being suppoled, as before, three Minutes and a half, there remains only one Minute to be accounted for, in computing the Height of the Sun, which is accordingly subtracted in the third Column from the Altitudes found by Computation. The Watch now appeared to be 9' 30" too slow.

Altitudes observed by Mr. *John Hadley*.

Time by Watch.	True Time.	True Alt. of the Sun's upper limb.	Refract. add.	App't Alt. of the Sun's upper limb.	Alt. of the Sun's upper limb observed.	Errors in the Division.	Observed Altitude of the Sun's upper limb corrected.	Errors of Observation.
9 52	55 10	02 25 36	52 54 11	36 54 05	36 46 00	0 00	45 00	-9 05
10 02	07 11	37 37 43	37 09 37	44 46 37	44 00	1 00	43 00	-1 46
06 00	15 30	38 04 00	38 08 38	05 08 38	04 00	1 00	03 00	-2 08
08 53	18 23	19 00	19 08	20 08	22 00	1 00	21 00	+0 52
12 25	21 55	36 25	36 07	37 32	41 00	1 30	39 30	+1 58
16 30	26 00	56 09	56 06	57 15	59 00	1 30	58 30	+1 15
18 50	28 20	39 07	39 06	08 13	06 00	0 30	05 30	-2 43
20 40	30 10	15 26	15 06	16 32	14 00	0 30	13 30	-3 02

The

The same continued by Mr. Bradley.

Time by Watch.	True Time.	True Alt. of the Sun's upper Limb.	Refract. on upper Limb.	App't Alt. of the Sun's upper Limb.	Alt. of the Sun's upper Limb observed.	Errors in the upper Division.	Observed Altitude of the Sun's upper Limb corrected.	Errors of Observation.
10 30 18	39 48 39	57 28 1	04 39	58 32 0	00 00	2 00	39 58 00	-0 32
33 23	42 53 40	10 02 1	04 40	11 06 0	12 00	2 00	40 10 00	-1 06
35 53	45 23	19 51 1	04	20 55 0	16 00	2 00	14 00	-6 55
37 48	47 18	27 13 1	03	28 16 0	32 00	2 30	29 30	+1 14
39 22	48 52	33 03 1	03	34 06 0	36 00	2 30	33 30	-0 36

Continued by Mr. Henry Hadley.

11 08 05	17 35 41	59 56 1	00 42	00 56 42	15 00	2 30	12 30	+11 14
16 20	25 50 42	16 56 1	00	17 36 0	20 00	2 50	17 30	-00 06
22 00	31 30	23 48 0	59	26 47 0	31 00	2 45	28 15	+01 28
24 20	33 50	29 35 0	59	30 54 0	34 00	2 45	31 15	+00 41
28 00	37 30	34 27 0	59	35 26 0	39 00	2 45	36 15	+00 49
33 45	43 15	40 32 0	58	41 30 0	45 00	3 00	42 00	+00 30
37 45	47 15	45 43 0	58	44 41 0	48 00	3 00	45 00	+00 19
40 50	50 00	45 25 0	58	46 23 0	49 00	3 00	46 00	-00 23
43 00	52 30	46 34 0	58	47 32 0	51 00	3 00	48 00	+00 28
47 00	56 30	47 42 0	58	48 40 0	52 00	3 00	49 00	+00 20
again.								
again.	12 00 00	48 02 0	58	49 00 0	52 00	3 00	49 00	+00 00

Between each of the five last of these Observations the Index was removed so as to make them entirely independent of one another; and from their near Agreement among themselves, and with good Part of the preceding, I conclude the true

Height of the Sun's Center above the real Horizon at Noon was exactly enough $42^{\circ} 33'$ his Semidiameter being 16 Min. from which, and the Sun's Declination $4^{\circ} 1'$ the Latitude of the Place will be $51^{\circ} 28'$, which is accordingly used in all the Computations.

Altitudes of the Sun observed *September 1, 1732*, Afternoon, near the *Bay of the Spile*, and under Sail Westward, by the Wooden Instrument forwards, the second Speculum remaining displaced as in the Morning.

Altitudes observed by Mr. Bradley.

Time by Watch.	True Time.	True Alt. of the Sun's lower Limb from the visible Horizon.	Refracti add.	App't Alt. of the Sun's lower Limb from the visible Ho- rizon.	Altitude of the Sun's lower Limb observed.	Errors of the In- strument's subtrah'd.	Altitude of the Sun's lower Limb corrected.	Errors of Observa- tion.	
12 07 30	12 17 00	42 12	13 11	00 42	13 13 43	20 00	1 6 0	42 14 00	+ 0 47
08 30	18 00	11 09	11 00	12 09	19 09	1 6 0	13 00	+ 0 51	
12 00	21 30	07 18	11 00	08 18	13 00	1 6 0	07 00	- 1 18	
19 50	29 20	41 56	00 11	00 41	57 00	01 00	1 6 0	41 55 00	- 2 00

The fame continued by Mr. Henry Hadley.

1 00 00	1 09 30	40 09	21 03	40 10	24 41	13 00	1 6 0	40 07 00	- 3 24	uncertain
01 35	11 05	03 20	11 04	04 24	10 00	1 6 0	04 00	- 0 24		
03 02	12 32	39 57	59 04	39 59	03 03	04 00	1 6 0	39 58 00	- 1 03	
04 21	13 51	52 57	11 04	54 01	01 02	00 1 6 0	56 00	+ 1 59		
06 14	15 44	45 31	11 04	46 35	40 52	00 1 6 0	46 00	- 1 35		
07 30	17 00	40 28	11 04	41 32	49 00	1 6 0	43 00	- 2 42		uncertain
08 45	18 15	35 37	11 05	36 42	40 00	1 6 0	34 00	- 0 27		
10 00	19 30	30 28	11 05	31 33	38 00	1 6 0	32 00	+ 0 27		
11 29	20 59	24 21	11 06	25 28	34 00	1 6 0	28 00	+ 2 32		
14 25	23 53	11 46	11 06	12 52	18 00	1 6 0	12 00	- 0 52		

The first and sixth Columns of the preceding Tables of Observations are copied from the Minutes as they were set down at the Time. The Divisions of the Wooden Instrument being not exact, I found it necessary to make a Table to correct them by, which was done partly by measuring with Compasses, and partly by examining them against those of another Instrument. The Corrections are every where to be substracted from the Angles observed, and the Errors of a Degree and three Minutes and a half, occasioned by the misplacing the second Speculum in all the forward Observations of *September* the 1st, being of the same kind, are joined with them, in the seventh Column of the Tables of those Observations. The last Column contains the Differences between the observed Altitudes, corrected by the forementioned Table, and the Altitudes as they ought to have appeared by the Computations. Among them there are two or three which so much exceed any of the rest, that for that reason they seem to be rather owing to Mistakes, in counting the Minutes on the Instrument, or the Time by the Watch, than to the Errors of the Observations.

The greatest Part of the Altitudes were taken by a Horizon not clear of Land, and by that Means not always so readily distinguishable. The Observers were all Persons quite unaccustomed to the Motion of a Ship at Sea, which in this Case was generally very great and quick, the Vessel we were in being only of about 60 Tuns Burthen, as the Master informed us, the smallness of which made it also more liable to be lifted up and let down again by the Waves: And if the Difference of Height occasioned by that Means was about four or five Feet, as we judged it to be, it

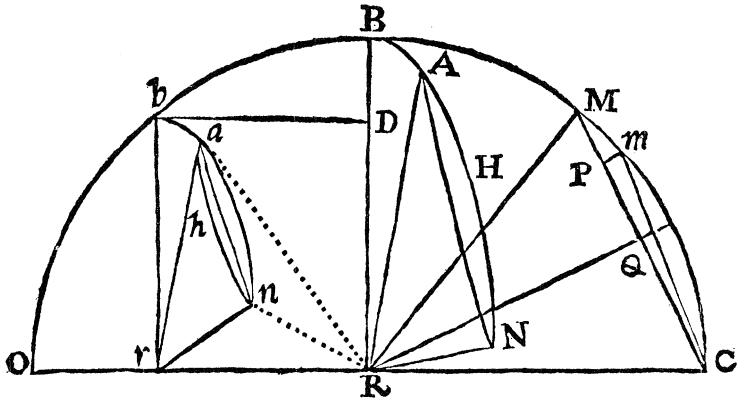
must necessarily sink and raise the visible Horizon by Turns near one Minute. The Computations of the Sun's Altitudes are all made for the Latitude of $51^{\circ} 28'$, whereas a good Part of them were taken under Sail, and upon different Tacks, the Vessel sometimes standing North East or North, and at other times South East, for near a quarter of an Hour at a time.

Several of these Circumstances may probably have contributed to increase the Inconsistency of the Observations; but as no particular Notice was taken of them at the Time, I content my self with barely mentioning them.

P O S T S C R I P T.

The Principle on which the Contrivance of this Instrument depends, was laid down in the before-mentioned *Philos. Transf.* Numb. 420. in one Proposition, and several *Corollaries*, the fifth of which contains the Grounds of an Approximation for correcting some small Errors which will arise if the Plane of the Instrument be suffered to vary too much from the great Circle passing through the two Objects, when the Observation is taken. There appears reason to think, that there will be very little Occasion in Practice for that Correction; but it was necessary to mention it, in order to explain the Nature of the Instrument; and as the manner of deducing that Corollary from the Proposition may not appear obvious to every Reader, I have here annexed the Demonstration of it.

Let



Let OBC in the annexed Figure represent an infinite Sphere, at whose Center R are placed the two Specula inclined to one another in any given Angle, and let their common Section coincide with the Diameter ORC . Let BAN be the Circumference of a great Circle, to the Plane of which the common Section of the Specula ORC is perpendicular, and BR its Radius: Let ban be the Circumference of a Circle parallel to BAN , and at the Distance from it Bb : Draw bD the Sine, and br the Sine complement of the Arch Bb : BD is the versed Sine of the same. Let A be a Point of an Object placed in the Circumference of the great Circle BAN , and N the Point in which its Image is formed by the two successive Reflections, as before described; and let a be a Point of another Object placed any where in the Circumference of the Parallel ban , and n its Image; and let abn be an Arch of a great Circle passing through the Points a and n . The Point a is at the same Distance from the great Circle BAN , as the Point b ,

i. e.

i. e. at the Distance Bb . Draw AR , AN , RN , ar , an , rn , aR and nR .

By the fourth Corollary the Figures ARN and arn are similar, and consequently the Line AN is to the Line an as AR or BR is to ar or br , *i. e.* as the Radius is to the Sine complement of the Distance Bb . But AN is the Chord of the Arch AHN of the great Circle BAN equal to the Translation of the Point A , or double the Inclination of the Specula, and an is the Chord of the Arch abn of a great Circle, measuring the Angle aRn , by which the Point a appears removed by the two Reflections, to an Eye placed in the Center R . Therefore the Translation, or apparent Change of Place of the Point a is measured by an Arch of a great Circle, whose Chord is to the Chord of the Arch AHN (equal to double the Inclination of the Specula) as the Sine complement of its Distance from the great Circle BAN is to the Radius.

From any Point C of the Circumference $OB C$ draw the Chords CM and Cm , to the same Side of the Point C , and equal to the Chords AN and an respectively, draw the Radius RM , and from R and m draw RQ and mP , both perpendicular to CM , and cutting it in Q and P . RQ is the Sine complement, and CM double the Sine of half the Angle MRC , or ARN , or of the Angle of Inclination of the Specula. The little Arch Mm will represent the Difference of the apparent Translations of the Objects in A and a ; and if it be very small, may be looked on as a strait Line, and the little mix'd Triangle MmP as a rectilinear one, which will be similar to RMQ , because RM is perpendicular to Mm and RQ

R Q to CM, and the Angles at Q and P right Angles. The Line CP may be taken as equal to Cm, and MP as the Difference of the Lines CM and Cm. Therefore the little Arch Mm is to the Line MP nearly as RM to RQ: But CM (*i. e.* AN) was to Cm (*i. e.* an) as BR to br, and the Difference MP of CM and Cm to the Difference BD of BR and br as CM to BR. Therefore Mm, the Difference of the apparent Translations, is to BD, the versed Sine of the Distance Bb, or to an Arch equal to it, in the compound Ratio of RM the Radius to RQ the Sine complement of the Angle of Inclination of the Specula, and CM double the Sine of the same to BR the Radius, *i. e.* as CM to RQ.

The Observation may be corrected by one easy Operation in Trigonometry, as will appear from the first Part of this Corollary, *viz.* by taking the half of the Angle observed, and then finding another Angle, whose Sine is to the Sine of that half, as the Sine complement of the Distance Bb is to the Radius: This Angle doubled, will be the true Distance of the Objects. But as this Operation, though easy, will require the use of Figures, I rather chose the Method of Approximation, because by that the Observer, retaining in his Memory the Proportions of the Sines of a few particular Arches to the Radius, may easily estimate the Correction without Figures, when the Angle is not great, and by a Line of artificial Numbers and Sines, may always determine it with greater Exactness than will ever be necessary.

When the Angle observed is very near 180 Degrees, the Correction may be omitted; for then it will be
easy

easy to keep the Plane of the Instrument so near that of the before-mentioned great Circle as not to want any, if the Situation of that Circle be known: If it be not, the Observer, when he sees the two Objects together, may turn the Instrument on the Axis of the Telescope, 'till he finds that Position of it by which he obtains the least Angle; and this (if the Specula are set truly perpendicular to the Plane of the Instrument) will always happen when the Objects appear to coincide in the Line $g b$, as expressed in the third Fig. of *Transf.* No 420.

In Page 152 of the same, a Rule is given for finding to which Hand of the Observer the Object seen by Reflection ought to lie, but is restrained to the particular Form of the Instrument there described. The general Rule is, that when the Index is brought to the beginning of the Scale (*i. e.* to 0° when the Instrument is designed for Angles under 90° , or to 90° when it is designed for Angles from 90° to 180°) if then a Line be imagined to be drawn on it parallel to the Axis of the Telescope, or Line of Direction of the Sight, so as to point towards the Object seen directly; which ever way this Line is carried by the Motion of the Index along the Arch from 0° towards 90° in the first Case, or from 90° towards 180° in the second, the same way the Object seen by Reflection ought to lie from that which is seen directly.

Erratum, Pag. 154. Line 24, read, *They may be either of Metal, or Glass Plates foil'd, having their two Surfaces, &c.*